Development of an analytical model to predict the permeability of fibrous media for air filtration

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\section{Introduction}

The determination of the permeability of fibrous media used for aerosol filtration is important as it influences the filter performances (pressure drop and collection efficiency). This is the reason why many models have been developed especially to predict the permeability of these soft non-woven materials. The permeability depends on the media structural properties: thickness, porosity, fiber material, size distribution and arrangement. Due to their processing history fibrous porous media are generally rather anisotropic, and in-plane and through-plane components of the permeability can be distinguished. The aim of this study is thus to propose an analytical model taking into consideration the anisotropic character of fibrous media, based on a mathematical weighted average procedure. This model is applied to determine the through-plane permeability of two different fibrous media used for particle filtration, and compared to experimental values obtained in this study as well as to eight models of the literature among which the classical ones of Davies \cite{1} and Jackson and James \cite{2} are presented here.

\section{The Representative Unit Cell models tested}

The manufacturing of non-woven fibrous media generally leads to 3D anisotropic structures yielding different in-plane and through-plane permeability components. Concerning fibrous media used for particle filtration, the flow mostly penetrates the filter through pathlines normal to the media. That is why for this kind of media the permeability is generally determined thanks to pressure drop measurements in flat configuration normal to the flow leading to the determination of the through-plane permeability. Here the permeability predicted by two rectangular Representative Unit Cell (RUC) models are compared to the experimental permeability values. They are based on determining the contribution of the resistance to the flow generated by fibers depending on their orientation with respect to the flow. In the first model, named 3D isotropic and which is represented on the left hand side in Figure 1, the flow is always parallel to one fiber and perpendicular to two fibers. A weight of 1/3 is thus assigned to parallel flow and a weight of 2/3 to transverse flow, to give the following expression for the permeability $B$, with $\varepsilon$ and $D_f$ being respectively the porosity and the fiber diameter:

\[
\frac{B}{D_f^2} = \left( \frac{48(1-\varepsilon)^2}{3\varepsilon^3} + \frac{15(1-\varepsilon)^{3/2}}{3\sqrt{(1-\varepsilon)}} \right)^{-1}
\]

(1)

Regarding these non-woven fibrous media as random distribution of fibers in parallel planes the second model, named 2D anisotropic, consists of two fibers (although connected) with respect to which the flow is perpendicular and no fiber with respect to which the flow is parallel. The through plane permeability of such representation is thus calculated by attributing a weight of 2/3 to flow perpendicular to the fibers and a weight of 0/3 to parallel flow:
\[ \frac{B}{D_t^2} = \left( \frac{2}{3} \left(1 - \epsilon \right)^{3/2} \right)^{-1} \]  

Figure 1: 3D isotropic RUC (left) and 2D anisotropic RUC (right).

3 Experimental determination of the permeability of two fibrous media

Two different fibrous media were considered. Their properties are given in Table 1. The mean fiber diameter was measured from analysis of pictures obtained by SEM observations (JEOL 5800-LV), and the porosity was determined by mercury porosimetry (Micromeritics Autopor IV). In Table 1 \( t \) is the media thickness. In order to determine the permeability of both media pressure drop measurements were carried out in flat configuration. The permeability was derived from the linear part of the curve representing the pressure drop versus the air velocity using the Darcy relationship.

<table>
<thead>
<tr>
<th>Media</th>
<th>Fibers material</th>
<th>( t ) (mm)</th>
<th>( D_f ) (( \mu m ))</th>
<th>( \varepsilon ) (%)</th>
<th>( B ) (m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G4</td>
<td>Cotton/Polyester</td>
<td>2.8±0.07</td>
<td>13.8±4.4</td>
<td>0.92±0.01</td>
<td>2.02.10⁻⁹</td>
</tr>
<tr>
<td>Fibrous support</td>
<td>Polyester</td>
<td>0.5±0.05</td>
<td>17.8±1.1</td>
<td>0.85±0.01</td>
<td>4.45.10⁻¹⁰</td>
</tr>
</tbody>
</table>

Table 1: Properties of the fibrous media studied.

4 Comparison between experimental permeabilities and predicted ones

The application of the models of Davies [1] and Jackson and James [2] under-predicts the permeability of the media by a factor of 15 for the G4 media, and 6 for the fibrous support. The 3D isotropic RUC model also leads to an under-prediction of the permeability for both media, but with a lower order of magnitude: the \( \frac{B_{\text{experimental}}}{B_{\text{modeled}}} \) ratio is respectively 8 and 5 for the G4 media and the fibrous support. The through-plane permeability calculation obtained with the 2D anisotropic model gives the best results: \( \frac{B_{\text{experimental}}}{B_{\text{modeled}}} \) ratios of 5 and 3 are obtained respectively for the G4 media and the fibrous support. This deviation is however not so important when regarding the sensitivity of the models to the porosity, and the experimental permeability based on the media thickness.

Conclusion

A new analytical 2D anistropic model has been developed to predict the through-plane permeability of non-woven fibrous media used for particle filtration, based on the assumption of random distribution of fibers in parallel planes. This model better predicts the permeability compared to other classical approaches, but there is still a discrepancy between measured and predicted permeability. This deviation is analysed regarding the sensitivity of both values to the media structural properties which must be determined with accuracy.

References


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