Identification of particles transport regimes in closed channel flow at low Reynolds numbers

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Keywords: Particles transport, Closed channel flow, Fluid-solid interaction

1. Introduction

Understanding the transport and deposition of inertial particles in channel flows is of fundamental importance in many environmental issues, such as underground pollution, as well as in several industrial applications, like water filtration. The present work aims to study the transport and deposition of non-Brownian particles in closed wavy and straight channel flows. The particles are small, spherical and have a low inertia and the flow is assumed to be steady and to follow the local cubic law. The studied domain is defined in a (X, Z) referential system where X is the horizontal direction corresponding to the main flow direction and Z the vertical direction along which gravity applies.

2. Theoretical framework

2.1. Particle motion equation

The equation describing the motion of a solid spherical particle of radius a and density ρ_p moving in a fluid of density ρ_f, kinematic viscosity ν and mean velocity U_0, as derived by Maxey and Riley [1], can be reduced to :

$$\frac{L_0}{U_0}\frac{d\vec{V_p}}{dt} = \frac{1}{\tau}(\vec{V_f} - \vec{V_p} + \frac{2}{9}\frac{a^2(1-k)g}{\nu}\frac{\vec{g}}{|g|}) + \frac{L_0}{U_0}\frac{3R}{2}\frac{D}{Dt}(\vec{V_f}) \tag{1}$$

with $\vec{g}$ the gravitational acceleration, $\vec{V_p}$ and $\vec{V_f}$ the particle and fluid velocities, L_0 the flow characteristic length, and $\tau = \frac{2}{9R}\frac{a^2 U_0}{\nu L_0}$ the particle response time. $R = \frac{2\rho_f}{2\rho_p + \rho_f} = \frac{2}{2k+1}$ is a dimensionless number with $k = \frac{\rho_p}{\rho_f}$ the ratio of particle density to fluid density. τ is the parameter that defines particle inertia. Since particles with low inertia are considered here (*i.e.* $\tau << 1$), equation 1 can thus be rewritten such as:

$$\vec{V_p} = \vec{V_f} + \frac{2}{9}\frac{a^2(1-k)g}{\nu}\frac{\vec{g}}{|g|} \tag{2}$$

2.2. LCL flow in a closed channel

The fluid flows in a closed channel of total length L_∞ and mean aperture H_0 defined by two horizontal periodic wavy walls with the same wavelength L_0. We assume that $\epsilon = \dfrac{H_0}{L_0} << 1$ and that the flow is laminar so that the inertia of the fluid can be neglected. The channel flow can then be described by the local cubic law (LCL) as defined by [2].

Under the LCL assumption, the components of the fluid velocity vector are given by:

$$V_f^x = \frac{3H_0V_0}{4H(X)}(1-\eta^2) \text{ and } V_f^z = \frac{3H_0V_0(\Phi(X) + \eta H(X))}{4H(X)}(1-\eta^2) \tag{3}$$

where $H(X)$ and $\Phi(X)$ correspond respectively to the variations of the channel aperture and of the channel middle line along the flow direction and $\eta = \dfrac{Z - \Phi(X)}{H(X)}$ is a cross channel coordinate. V_0 is the mean velocity of the flow which can be computed directly from the volumetric flow rate per unit length Q such that $V_0 = \frac{Q}{H_0}$.

2.3. Particle trajectory equation in closed channel flow

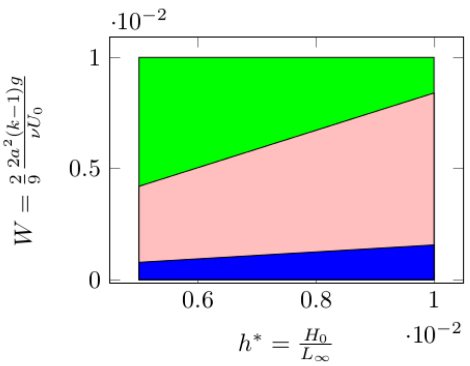
Combining equations 2 and 3, the trajectory of a particle in a closed channel LCL flow is thus defined by :

$$\frac{dZ_p}{dX_p} = \frac{-4WH(X)}{3H_0(1-\eta^2)} + (\Phi'(X) + \eta H'(X)) \tag{4}$$

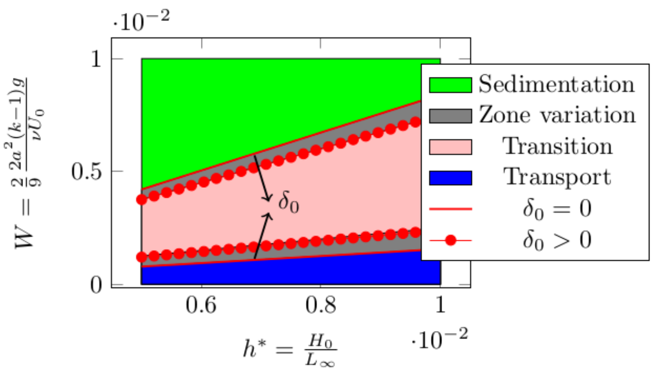
with $W = \frac{2}{9}\frac{a^2(k-1)g}{\nu V_0}$ a dimensionless number that represents the ratio between the particle sedimentation Stokes velocity and the flow mean velocity. Finally, the trajectory of the particle depends solely on the channel geometrical parameters $H(X)$ and $\Phi(X)$, and on W.

3. Regimes diagrams

Three different transport regimes can be defined for a group of particles entering the channel: (*i*) a transport regime where 75% of the particles exits the channel, (*ii*) a sedimentation regime where 75% of the particles settle inside the channel and (*iii*) a transition regime for which only a part of the particles (between 25 % and 75 % of particles) exits the channel. Using equation 4, we found that the transition between these different regimes is characterized by a linear variation of W as a function of the geometrical parameter $h^* = H_0/L_\infty$. A diagram of transport regimes could thus be established (Figure 1).



(a) Straight channel case



(b) Wavy channel case

Figure 1: Transport regimes diagram for weakly inertial particles in a straight channel (a) and in a channel with wavy walls (b)

Figure 1 (a) shows the transport diagram obtained for a straight channel, and Figure 1 (b) the diagram obtained for a channel with wavy walls. The effect induced by an increase of the walls mean amplitude A_0 (represented by an increase of the dimensionless parameter $\delta_0 = A_0/L_0$) is illustrated in Figure 1 (b), showing that the transport and sedimentation zones are wider in the case of a wavy channel.

A series of numerical simulations were conducted using COMSOL Multiphysics to investigate the influence of every parameters defining the channel geometry and the results compared with the analytical predictions. The comparisons confirmed the validity of the diagrams that could thus be used to predict the behavior of particles in channel flows.

References

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[2] R. W. Zimmerman and G. S. Bodvarsson, Hydraulic conductivity of rock fractures, *Journal of Fluid Mechanics*, Vol. 197, 1-30 (1996).