

The percolation threshold of fracture networks

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1. Introduction

Considerable progress with respect to previous works [1]-[4]. has been made in the investigation of percolation in fracture networks. Determination of the percolation threshold by direct numerical simulations has been conducted for a very wide range of regular, irregular and random fracture shapes, in monodisperse networks or in polydisperse networks containing fractures with different shapes and/or sizes. A greatly improved accuracy is achieved by using much larger domain sizes and extensive statistical data sets. The results are rationalized and a model involving a new shape factor is proposed, which accounts for the influence of the fracture shapes with an extremely good accuracy. A heuristic argument for the prediction of the percolation threshold [5] is revisited, and shown to provide good predictions, when appropriately adapted and when the fracture shapes do not depart too strongly from circularity.

The networks are composed of randomly located and oriented fractures in 3d space. The fractures are flat, 2d objects with convex contours. The investigated shapes are illustrated in Fig.1, where an example of networks of random quadrilateral is also shown. This sample is very small, for readability. The samples for the actual computations contain $\mathcal{O}(10^6)$ fractures.

For a given sample size L and network density ρ , the percolation probability $\Pi_p(L, \rho)$ is measured by checking the percolation status of a set random network realizations (typically 500). This is done for a range of density values (typically 20) and the density $\rho_c(L)$ which corresponds to $\Pi_p(L) = 1/2$ is determined by fitting the data by an erfc function. This is repeated for cells of increasing sizes, and an extrapolation to $L \rightarrow \infty$ is performed to eliminate the finite size effects, based on the successive $\rho_c(L)$ (although the largest cells are so large that the extrapolation only marginally affects the result).

The results are expressed in terms of the dimensionless density ρ' based on the excluded volume V_{ex} , or of its generalization by statistical averaging ρ'_3 in the case of polydisperse networks,

$$\rho' = \rho V_{ex}, \quad V_{ex} = \frac{1}{2} A P, \quad \rho'_3 = \rho \frac{1}{2} \langle A P \rangle \quad (1)$$

where A and P are the fracture area and perimeter. In terms of ρ'_c or $\rho'_{3,c}$, all the data for the percolation thresholds are believed to be accurate within at most ± 0.01 .

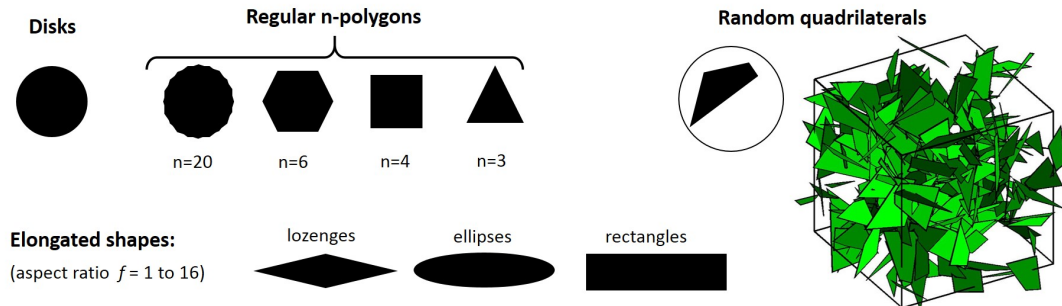


Figure 1: Illustration of the investigated fracture shapes, and example of a network containing 238 random quadrilaterals.

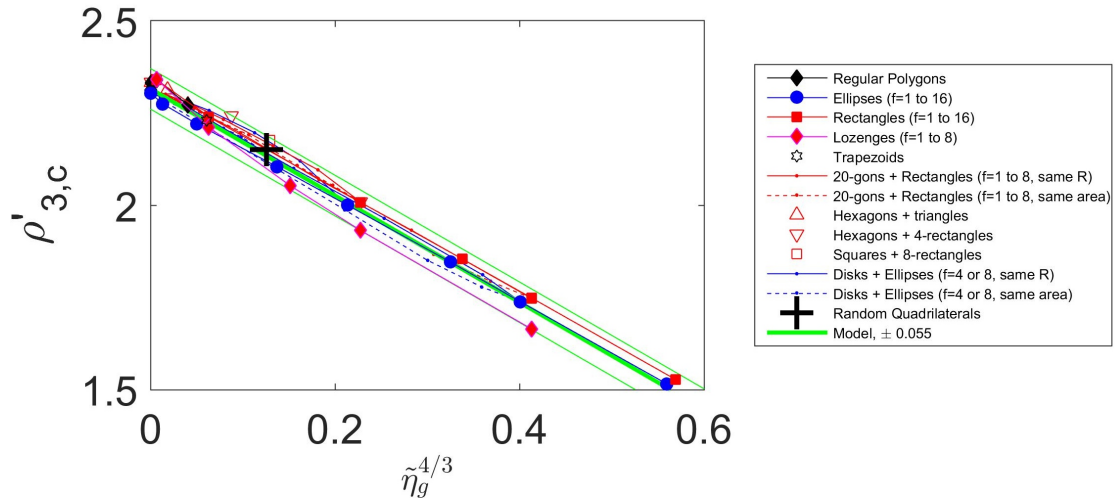


Figure 2: The percolation thresholds $\rho'_{3,c}$ for networks of fractures with all the shapes in Fig.1 and for mixtures of such fractures in various proportions as functions of the shape factor $\tilde{\eta}_g$.

2. Results

Two major trends can be observed. The percolation threshold $\rho'_{3,c}$ decreases from its value 2.30 for monodisperse disks when the fracture shape departs from circularity, whereas it increases with size polydispersity. The thresholds for networks of fractures with all the shapes in Fig.1 and for mixtures of such fractures in various proportions are plotted in Fig.2 as functions of $\tilde{\eta}_g$. This shape factor involves the fracture giration radius R_g , it ranges from 0 for disks to 1 for very elongated shapes, and it should be taken in RMS statistical average weighted by AP in the case of polydisperse networks,

$$\tilde{\eta}_g = 1 - \sqrt{\frac{A}{2\pi R_g^2}}, \quad \text{or} \quad \tilde{\eta}_g = \left\{ \frac{\langle AP \left(1 - \sqrt{A/2\pi R_g^2} \right)^2 \rangle}{\langle AP \rangle} \right\}^{1/2} \quad (2)$$

Note that both the shapes and sizes of the fractures in the networks of random quadrilaterals are strongly polydisperse (see Fig.1). All the data are represented within ± 0.055 by the model

$$\rho'_{c,r} = 2.315 \left[1 - \frac{5}{8} \tilde{\eta}_g^{4/3} \right] \quad (3)$$

Hence, the shape factor $\tilde{\eta}_g$ and the model (3) capture with a very good accuracy the effect of the fracture shape and in some respect of the size distribution. The analysis of the influence a strong size polydispersity is still underway.

Furthermore, a heuristic argument [5] for the prediction of the percolation threshold has been revisited and adapted. Its application requires the determination of the mean center-to-center distance of intersecting fractures, for which an approximate but fairly accurate analytical expression could be obtained. This argument yields good predictions of $\rho'_{3,c}$ when the fracture shapes do not depart too strongly from circularity. This result may provide an avenue for other applications in continuum percolation problems.

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