A comparative study of five boundary treatment schemes for stationary complex boundaries in the lattice Boltzmann method

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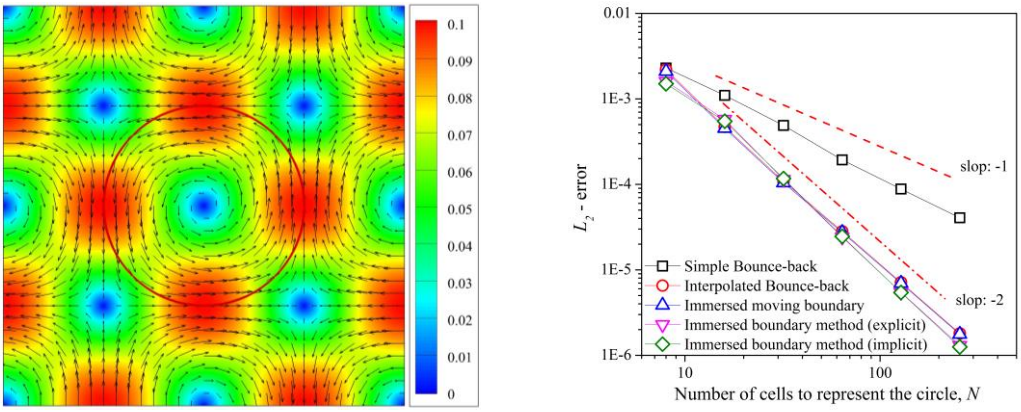
1 Introduction

In recent years, the lattice Boltzmann (LB) method has become a promising approach for direct numerical simulations of simple and complex fluids and flows, such as particulate flows and flows through porous media [1]. A key issue in the LB simulations of complex flows is concerned with the accurate treatment of fluid-solid boundaries. In this study, a comparative study with five widely used schemes for the treatment of stationary complex boundaries is conducted to assess their accuracy, difficulty of implementation and computational efficiency. The five schemes are the simple bounce-back (SBB), an interpolated bounce-back (IBB)[2], two direct-forcing immersed boundary methods (IBMs) [3] and an immersed moving boundary method (IMBM) [4]. The two IBMs include an explicit and an implicit diffusive interface scheme (IBM-ED, IBM-ID). The Bhatnagar-Gross-Krook (BGK) model is used for the collision term. We first carried out simulations for the two-dimensional (2D) Taylor-Green decaying vortex flow. The analytical solution for this problem is used to determine the accuracy order of different schemes. Then the flow past a circular cylinder in a confined channel with the Reynolds number Re = 20 and 100, corresponding to steady and unsteady flow patterns, are considered. The available benchmark data are used to evaluate the results. Besides the overall information such as the drag and lift coefficients and the Strouhal number, we also specially considered some local criteria including the satisfaction of no-slip condition and the force distribution around the cylinder. We find that IBB leads to the most accurate results in terms of both overall and local criteria. The results of overall criteria with BB and IMBM show a little higher accuracy than with IBM-ED and IBM-ID with the same mesh size. However, the amplitudes of variation in hydrodynamic force distribution around the cylinder with BB and IMBM are much higher than with IBM-ED and IBM-ID. Though the IBM-ID shows a better satisfaction of no-slip condition around the cylinder than IBM-ED, the overall accuracy does not decrease much. Regarding the computational efficiency, IBB and BB are most superior, followed by IMBM and IBM-ED. The extra iteration with IBM-ID dramatically increases the computational cost. The present study may help researchers select an applicable boundary condition treatment scheme for their LBM.

2 Physical problems and results

2.1 *Two-dimensional Taylor-Green decaying vortex*

To evaluate the accuracy order of different schemes, the two-dimensional Taylor-Green vortex flow in a square box [5], which possesses an analytical solution, is considered as the first test problem. Figure 1a shows an example of the velocity solution, which is obtained with the interpolated bounce-back scheme using 160×160 lattice cells. In this case, we added an embedded circle to represent a boundary of complex geometry. Figure 1b presents the overall errors in terms of L_2 norm versus the number of lattice cells representing the domain. It is shown that SBB is first-order, and the other four schemes are second-order. In addition, the two immersed boundary methods show a bit higher accuracy than SBB and IMBS in the magnitudes of error norm.



(a) (b)

Figure 1: (a) Velocity magnitude and vector of the Taylor-Green vortex. The result is obtained with the interpolated bounce-back scheme with 160×160 lattice cells. The solid line represents an embedded circle. (b) Overall accuracy of different schemes for the Taylor-Green vortex case.

2.2 *Flow around a fixed two-dimensional circular cylinder in a confined channel*

We consider the 2D flow around a cylinder with circular cross-section in a channel as the second test case [6]. This is a benchmark problem which has been widely used for validation of numerical method. Note that the axis of the cylinder is not on the horizontal symmetry plane of the channel, so the flow is slightly asymmetric and therefore the lift force is no longer zero when the flow is steady. In Table 1, we summarized some characteristic parameters (including the drag coefficient C_D, the lift coefficient C_L, the pressure difference Δp) for different schemes on two grids characterized by the cylinder diameter, $D = 20$ and 40, for the case $Re = 20$. In addition, we also add the number of iterations N_t and CPU time to reach the steady state.

Scheme	C_D	C_L	Δp	*Nt*	*CPU*
D = 20					
SBB	5.7794	0.0140	0.1160	4530	114.43
IBB	5.5845	0.0104	0.0091	4530	116.87
IMBM	5.5577	0.0715	0.1120	4530	143.71
IBM, explicit	6.0316	0.0142	0.1052	4530	203.01
IBM, implicit	5.9869	0.0160	0.1052	4530	325.51
D = 40					
SBB	5.6635	0.9533	0.1174	6620	587.34
IBB	5.5550	0.0101	0.1137	6620	579.30
IMBM	5.5557	0.0091	0.1151	6630	632.55
IBM, explicit	5.8321	0.0118	0.1052	6370	832.57
IBM, implicit	5.7889	0.0130	0.1047	6620	1160.98
Ref. [6]	5.5700-5.5900	0.0104-0.0110	0.1172-0.1176		

Table 1: Some characteristic parameters for the flow past cylinder case with $Re = 20$.

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